Report _ Students' Mathematisation in Solving Mathematical Literacy Problems with Space and Shape Contents

by Lestariningsih -

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Students' Mathematisation in Solving Mathematical Literacy Problems with Space and Shape Contents

Students' Mathematisation in Solving Mathematical Literacy Problems with Space and Shape Contents

Lestariningsih[1], S M Amin[1], A Lukito[1], M Lutfianto[2]

[1]Department of Mathematics, Universitas Negeri Surabaya, Indonesia
[2]STKIP Al-Hikmah, Surabaya, Indonesia

Abstract. This paper aimed to describe four undergraduate students' mathematisation from Surabaya and Sidoarjo in solving mathematical literacy problems with space and shape contents. This study used descriptive research with a qualitative approach. Data were collected by giving the mathematical literacy problem with space and shape contents and interviews. Next, data were analysed by studying the four mathematisation processes: formulating, using, interpreting, and evaluating. The results of this study pointed out that students'' mathematisation in solving mathematical literacy problems with space and shape contents were as follows 1) formulating contextual problems into mathematical problems by verbally describing, using mathematical symbols that have been understood, and using appropriate modelling, 2) building or using direct mathematical objects (facts, concepts, principles, and procedures) and indirect (mathematical reasoning) to obtain mathematical solutions, 3) summarising the results obtained from the counting operations that have been performed, and the last 4)stating that the mathematical solution obtained is correct by raising logical reasons. Students' mathematisation was a principal process in solving mathematical literacy problem with space and shape contents, so it was important to emerge students' mathematisation in the mathematical classroom or daily activities.

1. Introduction

Problem solving skills in mathematics especially contextual mathematics problems require mathematisation. Mathematisation is not only used when someone makes a model or a mathematical representation of a problem with a real-life context but also when the process of solving the problem and interpreting it into a real-life context [1, 2]. Freudenthal [3] declares that mathematisation means making things more mathematical. Also, mathematisation is not only an act of mathematicians but also becomes the activities of learners in understanding the daily situations using a mathematical approach. Mathematisation is divided into two kinds, namely horizontal mathematisation and vertical mathematisation [4]. Mathematisation can be different for each learner in solving a problem.

Contextual problem is also used in Programme for International Student Assessment (PISA). Characteristics of problems which used in PISA is arranged in different domains require not only vertical mathematisation but also horizontal mathematisation. Mathematisation in PISA refers to Freudenthal's idea in Realistic Mathematics Education (RME) [5]. Some researchers conclude that mathematical cycles (mathematical modelling) in PISA are identical with horizontal mathematical and vertical mathematisation cycles in RME [6,7].

Mathematisation not only as part of the process of solving mathematical problems or contextual problems but also as part of a learning process and teaching practice which makes mathematisation capability to be owned by students [8,9]. But, many students have difficulty to do mathematisation when solving contextual problems. The students' mathematisation processes were not even and/or consistent even for the problem with same context topic [10].

Mathematisation proposed by Roux [11] is the application of concepts, procedures, and methods developed in mathematics to an object or knowledge in either mathematics or other disciplines. Meanwhile, Fosnot [12] says that mathematisation is a constructive process including observing patterns in special cases, analysing the reasons for something happening, stating in some form of generalisation, and looking for flexibility in strategy making or proof. Whereas OECD [13] stated that mathematisation can involve transforming a problem defined in the real world to a strictly mathematical form (which can include structuring, conceptualising, making assumptions, and/or formulating a model), or interpreting or evaluating a mathematical outcome or a mathematical model in relation to the original problem. Then, mathematisation in this study is the activity of transforming a problem expressed in a real life context into a mathematical model or representation, then the completion of a mathematical model or representation is interpreted into a real life context. The real world is defined as everything that is outside mathematics, like everyday life and the environment [14].

The steps of mathematisation proposed by Lynch [15] were (1) considering the real-world problem, (2) organizing based on mathematical contexts, (3) paring down to the most important aspects, (4) using mathematical skills and solving problem, and (5) considering the solution toward terms of the real situation. OECD [13] declared the steps of mathematisation which were (1). formulate, (2) employ, (3) interpret, and (4) evaluate. The steps of mathematisation which used in this study were (1) formulating a problem of real world context into mathematical problem, (2) using the facts, concepts, procedures and mathematical reasoning to obtain mathematical solution of the problem mathematically, (3) interpreting the mathematical solution to the real world context to the initial problem, and (4) evaluating solution to real world context to the problem.

Mathematisation is also one of the seven fundamental mathematical capabilities required by the individual in solving mathematical literacy problem [16]. Mathematical literacy problem is expressed mainly in the form of contextual problems or mathematical problems using real world context because of their structure emphasizes the need to develop the capacity of a person in using mathematics in a variety of contexts. Mathematical literacy problem can be used to measure the level of mathematical literacy that is owned by someone. Mathematical literacy describes a person's ability to reason mathematically and use the concepts, procedures, facts and mathematical tools to explain and predict phenomena. Mathematical literacy is an essential ability for young people to prepare for life in a modern society in the 21st century [16].

Problem in PISA is based on aspects of mathematical content and context. The context indicates the location or places that exist in the problem and mathematical content is a mathematical topic used in problems. There are four categories of content used in PISA, namely, (1) change and relationships, (2) shape and space, (3) quantity, (4) uncertainty and data [17]. This study uses shape and space content contained in PISA. Shape and space content include phenomena about the visual world that involves patterns, properties of objects, positions and orientations, representations of objects, encoding visual information, navigation, and dynamic interactions associated with real forms. We use shape and space content because this content includes aspects of mathematical content in mathematics present in the curriculum.

Previous study about mathematisation showed that student's specific forms of mathematisation. It was associated with the mathematisation of motions which were (1) deductive reasoning, (2) alternative mathematical methods, and (3) geometrical diagrams [11]. Other study about mathematisation of Irish students showed that their ability to mathematics (that is the ability to transfer a realistic situation into a mathematical situation) was poor hence there was no evidence of any significant gender difference in test performance, and they find difficulty when applying mathematical knowledge in unfamiliar situations [15].

Based on the previous explanation, the purpose of this study was to describe students' mathematisation in solving mathematical literacy problems with space and shape contents.

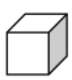
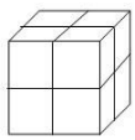
2. Method

This study used descriptive research with qualitative approaches. Participants in this study were four undergraduate students of mathematics education with high mathematics ability. The selection of these participants was made by using mathematics skills test. Before the mathematics skills tests were used, validity was performed to maintain the quality of the problems, namely construct validity, content validity, face validity, and item validity.

Then to know the ability of mathematisation, students were asked to solve mathematical literacy problems with space and shape content by adapting the PISA problems using Indonesian context (see Table 1). Problems were designed as two problems given at different times for triangulation needs. Mathematical literacy problems that have been designed to be validated through expert review include the suitability of content, construct, and face.

After students finished mathematical literacy problems, we conducted interviews to explore mathematisation data. Interview data were analysed using qualitative data analysis consist of three stages, namely data condensation, data presentation, and drawing conclusion.

Table 1. Mathematical literacy problems with space and shape contents.

First Problem	Second Problem
Susan wants to make a solid from small cubes. Here is a picture of a small cube (Figure 1) and a solid that build from small cubes (Figure 2). She uses glue to fit the cubes to be solid. First, Susan glued eight cubes to make a solid as shown in figure B. Then Susan wants to make a solid with a length of 6 small cubes, a width of 5 small cubes, and a height of 4 small cubes. She wants to use as little as possible a small cube with a hole in the solid. How many cubes are Susan needed to make this? Explain!   **Figure1.** Small Cube.**Figure 2**. Solid.	A cake trader "*TerangBulan*" sells two kinds of *terangbulan* with the same thickness but different in sizes (See Figure 3). The small *terangbulan* has a diameter of 30 cm, and the price is Rp 30.000, - and the big *terangbulan* has a diameter of 40 cm, and the price is Rp 40.000,-. Which *terangbulan* is cheaper? Explain!  **Figure 3**. *TerangBulan*.

3. Results and Discussion

3.1. Students' mathematisation on formulating a problem of real world context into a mathematical problem

Regarding to the first step, students identified the mathematical aspects of real life context in the problems by presenting what was known in the form of verbal descriptions. In the first problem, students wrote that Susan wanted to make the solid that looked likea solid object in the inner part (see Figure 4) while in the second problem, the students wrote the diameter and the price of the small and big *terangbulan* (see Figure 5).

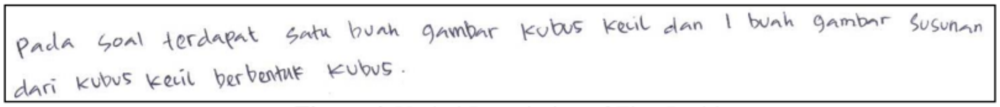


Figure 4. Verbal Description of First Problem.

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Furthermore, students also identified the important variables according to real situation problems by stating the image information in the problem using mathematical statements or symbols that have been understood. In the first test, students declared information in the form of Figure 1 (Small Cubes) and Figure 2 (Solid from Small Cubes) into a description that was on the problem there was one figure of small cube and 1 drawing the composition of a small cube shaped block while in the second problem. Students declared the information that was obtained by writing the small *terangbulan* as the first *terangbulan* and the big *terangbulan* as the second *terangbulan*. Figure 5 also illustrates those students using symbols in the second problem.

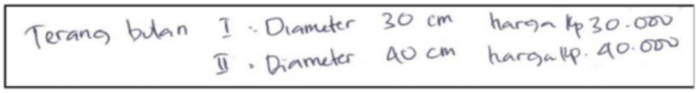


Figure 5. Verbal Description of the Second Problem.

Then, students represented the situation mathematically by using appropriate modelling to clarify the problem and facilitated themselves in solving the problem, as well as identified the variables that were important in the context of the problem to determine the mathematical aspects. In the first test, students made a visualization of the information. "*Susan wants to make a solid that looks like a solid with full of cubes includes in the inner part. It has length 6 small cubes, width 5 small cubes, and height 4 small cubes*" as a representation of a build composed of several small cubes to clarify the problem and facilitated them in solving the problem. In the second test, students made visualisation of small *terangbulan* and big *terangbulan* as the representation of the first circle and the second circle respectively.

Students identified the important variables in the context of the problem to determine the mathematical aspect of the first problem by calculating the length, width and height by marking using the "dots". Then, it was known that the length was 6 small cubes, the width was 5 small cubes, and the height was 4 small cubes (see Figure 6). They said, *"This one is 6 in length, 5 small cubes in width and 4 small cubes in height (while recalculating the dot on the drawing in the answer sheets)." While from the second problem, students knew that the radius is half of the diameter by stating, "The radius is half of the diameter, and counts the radius by* $30 \div 2 = 15$".

Moreover, from the data analysis, it was obtained that the main categories and other categories are seen as subcategories. The main category that emerges is the way students formulate mathematical literacy problem with the content of shape and space into mathematical problems with three subcategories, namely (1) students formulate mathematical literacy problem into mathematical problems by using verbal description, (2) students formulate mathematical literacy problem into mathematical problems using mathematical statements or symbols that have been understood, and (3) students formulate mathematical literacy problem into mathematical problems using the appropriate modelling to clarify the problem and facilitate themselves in solving the problem.

3.2. Students' mathematisation in using the facts, concepts, procedures and mathematical reasoning to obtain a mathematical solution of the problem mathematically

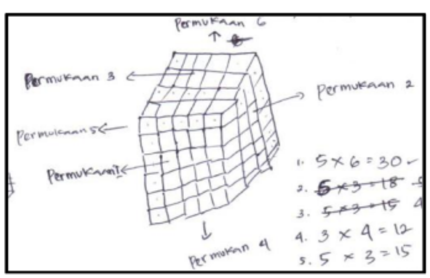
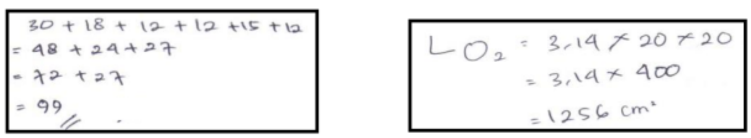


Figure 6. Solid from Cubes.

Regarding to the second step, students designed a strategy to find a mathematical solution by planning to use the concept of cube and circle arrangement to represent the context contained in the problem. Students also designed to apply principles that fit with the context of the problem, namely the surface area of solid and the area of the circle. For the second problem, they said, "*The area of the circle=* $\pi \times r^2$". Then, students planed strategies to find mathematical solutions by constructing facts by applying the concept of sequence when using labels to express the context contained in the problem. They used labels ranging from first surface to sixth surface on each constructed surface drawn respectively to represent the front surface, the right surface, the top surface, the lower surface, the left surface, and the back surface (see Figure 6). They said, " *In this problem, the front side is likened surface 1, hold the side here (right) is likened surface 2, top side is likened surface 3, bottom side is likened subsurface 4, and the left side is likened surface 5, and the back side is likened surface 6."*

Students used the concepts of cube, rectangle, and circle to represent the context contained in the problem. They also used the procedure of addition, multiplication, and division as well as doing mathematical reasoning (deductive reasoning using the associative properties of addition and multiplication) to obtain a mathematical solution. Figure 7 shows the mathematical reasoning mentioned.



Associative properties of addition associative properties of multiplication

Figure 7. The Associative Properties of Addition and Multiplication

In the step of using facts, concepts, principles, procedures and mathematical reasoning to derive mathematical solutions from mathematical problems, it was obtained a main category and two subcategories. The main category that emerged was students constructing or using mathematical learning objects to obtain mathematical solutions. Then, two subcategories of data could be created in the convergence interpretation. First, students constructed or used mathematical learning objects directly in mathematical literacy to obtain mathematical solutions. Second, students used indirect mathematics learning objects to obtain mathematical solutions that are by using mathematical reasoning like the associative properties of addition and multiplication.

3.3. Students' mathematisation on interpreting the mathematical solution to the real world context to the initial problem

In the step of interpreting mathematical solutions into real world contexts in the original problem, students concluded the results obtained from the counting operations that have been done on the first and second problem. In the first problem, they concluded by stating the number of small cubes needed to make the solid. They said, "*Susan will use 99 small cubes to make the solid block*." Then, in the second problem, the students concluded the results obtained from the counting operations that have been done by stating that *terangbulan* that cost cheaper was *terangbulan* which had size 40 cm. They said, "In conclusion, I think the cheaper *terangbulan* is 40 cm in size".

3.4. Students' mathematisation onevaluating solutions to real world context to the problem

In the last step, students evaluated the compatibility between mathematical solutions in the context of real world problems by stating that the mathematical solutions obtained were correct because students were already using the appropriate procedures. In the first problem, students counted the number of small cubes of each surface then added. In the second problem, students calculated the area of both *terangbulan* then used the comparison to get a mathematical solution. Furthermore, students explained the reasons why mathematical results or conclusions were appropriate or not appropriate to the context of the problem by stating that the results or conclusions obtained are suitable with the context of problem because students used concepts, principles, and procedures to find out. In the first problem, they said, *"The results or conclusions obtained are suitable with the context of problem because Wealready counted the number of small cubes on each side and sum up them"*. Then, in the second problem, they said, *"We used formula to get areas of circles then we compared them"*.

Moreover, findings on mathematisation of students with high mathematics ability in solving mathematical literacy was in line with study conducted by Lynch [15] because mathematical literacy problems was using realistic context for students. Students with high mathematics skills also did mathematisation by using facts, concepts, principles, procedures and mathematical reasoning to obtain mathematical solutions [15,16]. However, it was found that some important categories when students did mathematisation namely the way students formulate mathematical literacy problems using shape and space content into mathematical problems, and students constructing or using mathematical learning objects to get mathematical solutions.

4. Conclusions

It was important to highlight that mathematisation is a fundamental process in solving mathematical literacy problems because through mathematisation it can be known (1) the process of formulating real world context problems into mathematical problems, (2) the use of mathematical learning objects to obtain mathematical solutions, (3) the summary of the results, and (4) the evaluation of the results. The result of study showed that the students did the mathematisation by (1) identifying the mathematical aspect in real situation problem, (2) identifying/raising the important variables according to real situation problem, (3) representing the situation mathematically by using the appropriate modelling, (4) planning the strategy to find mathematical solutions, (5) applying mathematical facts/rules/algorithms/structures or mathematical strategies when looking for solutions, (6) interpreting mathematical solutions in real life context, (7) evaluating the fit between mathematical solutions in the context of real-world problems, and (8) explaining the reasons why results or mathematical conclusions are appropriate or not inappropriate with the context of the problem.

Mathematical literacy and PISA problems with change and relationships, quantity, uncertainty and data content were recommended to be used. It was recommended to develop a more complete description on students' mathematisation by applying various problems. Moreover, further studies could also use problems that were developed by researchers.

References

[1] Gravemeijer K and Terwel J 2000 *J. Curr. Studies* **32** 777
[2] Lestariningsih L, Amin SM, Lukito A andLutfianto M 2018 *Proc. of University of Muhammadiyah Malang's 1st INCOMED, ASSEHR* **160** p. 291
[3] Freudenthal H 1968 *Educational Studies in Mathematics* **1** 3
[4] Wake G 2014 *Educational Studies in Mathematics* **86** 271
[5] OECD 2003*The PISA 2003 Assessment Framework: Mathematics, Reading, Science and Problem Solving Knowledge and Skills*(Paris: OECD Publishing)
[6] Kaiser G and Willander T 2005 *Teaching Mathematics and Its Applications* **24** 48
[7] De Lange J 2006 *Tsukuba J. Educ. Study in Mathematics* **25**13
[8] Biccard P and Wessels D 2015 *Pythagoras* **36** 294
[9] Biccard P and Wessels D 2017 *African Journal of Research in Mathematics, Science and Technology Education* **21**61
[10] Murata A and Kattubadi 2012 The Journal of Mathematical Behavior **31** 15
[11] Roux S 2010*Early Science and Medicine* **15** 319
[12] Fosnot C T 2005 *The Constructivist* **16** 1
[13] OECD 2016 *PISA 2015 Assessment and Analytical Framework: Science, Reading, Mathematic and Financial Literacy* (Paris: OECD Publishing).

[14] Siregar N F 2016 *Logaritma* **4** 17
[15] Lynch P E 2011 *Mathematisation and Irish students: The ability of Irish second-level students to transfer mathematics from the classroom to solve authentic, real-life problems* (Doctoral dissertation, National University of Ireland Maynooth).
[16] Blum W and Ferri R B 2009 *J. Math. Modelling and Application* **1** 45
[17] Stacey K 2011 *J.* Math. *Educ. (Indo-MS_JME)* **2** 1

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